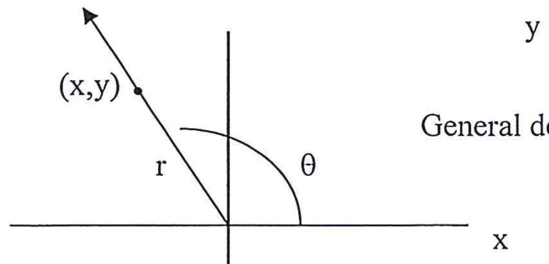
Great Truths of the Universe Plus:
Authored by Stephen Drake (June 16, 2014)



General definitions of the trigonometric operators:

$$\text{sine}(\theta) = \sin(\theta) = \frac{y}{r} \qquad \text{cosecant}(\theta) = \csc(\theta) = \frac{r}{y}$$

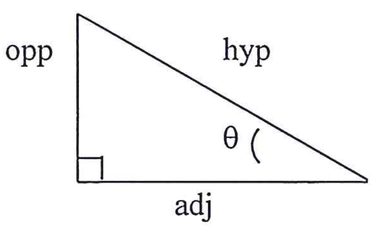
$$\text{cosine}(\theta) = \cos(\theta) = \frac{x}{r} \qquad \text{secant}(\theta) = \sec(\theta) = \frac{r}{x}$$

$$\text{tangent}(\theta) = \tan(\theta) = \frac{y}{x} \qquad \text{cotangent}(\theta) = \cot(\theta) = \frac{x}{y}$$

Seventeen "nice" angles:

0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°

Right triangle definitions for the three, primary trigonometric operators:

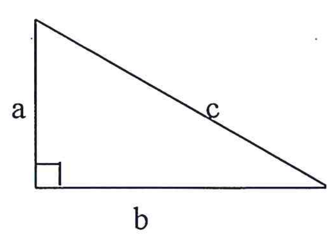


$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

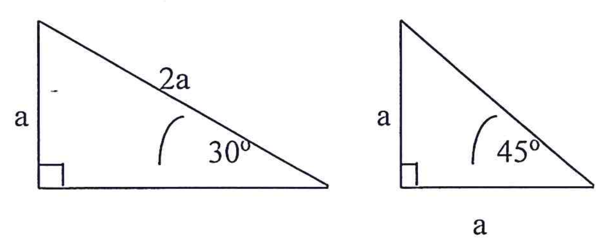
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

Pythagorean Theorem: Right triangles, only.



$$a^2 + b^2 = c^2$$

Special right triangles:



People did not invent mathematics. They discovered it and they keep on discovering more great truths of the universe.

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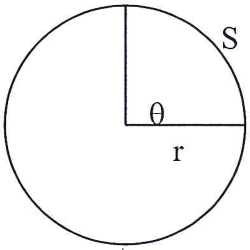
Radian Measure:



$S = r\theta$ where "θ" is a radian measure.

$360^{o} = 2\pi$

One radian is the angle subtended when one radius length is laid out on the circumference of a circle.

degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	und.	$-\sqrt{3}$	-1	$-1/\sqrt{3}$	0

degrees	180°	210°	225°	240°	270°	300°	315°	330°	360°
radians	π	$7\pi/6$	$5\pi/4$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$7\pi/4$	$11\pi/6$	2π
$\sin\theta$	0	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1	$-\sqrt{3}/2$	$-1/\sqrt{2}$	$-1/2$	0
$\cos\theta$	-1	$-\sqrt{3}/2$	$-1/\sqrt{2}$	$-1/2$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	und.	$-\sqrt{3}$	-1	$-1/\sqrt{3}$	0

Basic Identities

$\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
$\csc\theta = \dfrac{1}{\sin\theta}$	$\cos^2\theta + \sin^2\theta = 1$
$\cot\theta = \dfrac{1}{\tan\theta}$	$1 + \tan^2\theta = \sec^2\theta$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot^2\theta + 1 = \csc^2\theta$

"Knowing that something is wrong, but a person does it anyway; that person has committed a sin. In mathematics, find a new way to do something wrong -- at least that way a student can be awarded points for creativity."
Drake

"Do or do not. There is no try."
Yoda

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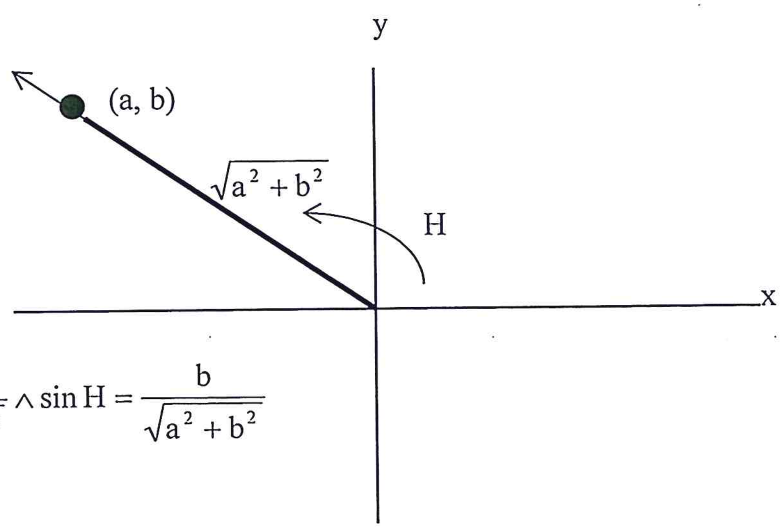
More Identities That Everyone Knows

$\cos(A+B)=\cos A\cos B-\sin A\sin B$	$\sin(A+B)=\sin A\cos B+\sin B\cos A$
$\cos(A-B)=\cos A\cos B+\sin A\sin B$	$\sin(A-B)=\sin A\cos B-\sin B\cos A$
$\cos(2B)=\cos^2 B-\sin^2 B$	$\sin(2B)=2\sin B\cos B$
$\cos(2B)=2\cos^2 B-1$	
$\cos(2B)=1-2\sin^2 B$	
$\cos\left(\frac{\theta}{2}\right)=\operatorname{sgn}\left(\cos\left(\frac{\theta}{2}\right)\right)\sqrt{\frac{1+\cos\theta}{2}}$	$\sin\left(\frac{\theta}{2}\right)=\operatorname{sgn}\left(\sin\left(\frac{\theta}{2}\right)\right)\sqrt{\frac{1-\cos\theta}{2}}$
$\cos^2\theta=\frac{1+\cos(2\theta)}{2}$	$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$

Be Aware of These Identities

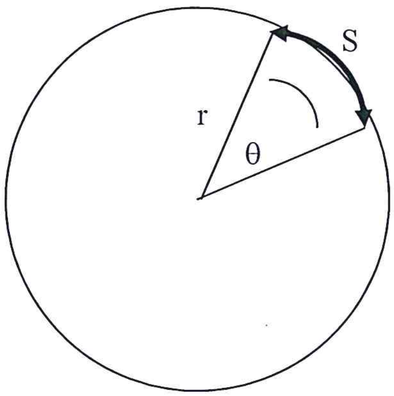
$\sin A\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$	$\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
$\cos A\sin B=\frac{1}{2}[\sin(A+B)-\sin(A-B)]$	$\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
$\cos A\cos B=\frac{1}{2}[\cos(A+B)+\cos(A-B)]$	$\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
$\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$	$\cos C-\cos D=-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$



$$\cos H=\frac{a}{\sqrt{a^2+b^2}}\wedge\sin H=\frac{b}{\sqrt{a^2+b^2}}$$

$$\therefore a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}\,\sin(\theta+H)$$

Aren't Great Truths wonderful?



$S = r\theta$ where "θ" is in radians.

$\frac{S}{t} = r\left(\frac{\theta}{t}\right)$ where "t" is time.

$\frac{S}{t} = v$ where "v" is linear velocity.

$\frac{\theta}{t} = \omega$ where "ω" angular velocity in radians/time.

$\therefore v = r\omega$

In calculus, we often use these modifide forms of the half – angle identities:
$\sin^2 x = \frac{1-\cos(2x)}{2}$ and $\cos^2 x = \frac{1+\cos(2x)}{2}$.

The problem with political jokes is that they often get elected.

> "Human mind and culture have developed a formal system of thought for recognizing, classifying, and exploiting patterns. We call it mathematics."
>
> Ian Stewart Nature's Numbers

"Do not worry about your difficulties in Mathematics. I can assure you mine are still greater."
Albert Einstein

"Only the universe and human capacity for ignorance are infinite. I'm not so certain about the universe."
Albert Einstein